

$$V_{m_f}^+ = -2\beta\ell + \beta\ell^2 + \frac{1}{2} \log(2\beta\ell) + \frac{3}{4}, \quad \ell \geq 1/(2\beta)$$